

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER-IV (NEW) EXAMINATION – SUMMER 2021****Subject Code:3140708****Date:07/09/2021****Subject Name:Discrete Mathematics****Time:02:30 PM TO 05:00 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		MARKS
<b>Q.1</b>	(a) Among 100 people at least how many of them were born in the same month?	<b>03</b>
	(b) Prove that: $(A \cup B)' \equiv A' \cap B'$ .	<b>04</b>
	(c) Define the following:	<b>07</b>
	1) Composition of functions	
	2) Monoid	
	3) Existential Quantifier	
	4) Partially Ordered Set	
<b>Q.2</b>	(a) Explain types of a Relation with a suitable example.	<b>03</b>
	(b) Rewrite the following statements using quantifier variables and predicate symbols:	<b>04</b>
	1) All birds can fly.	
	2) Some women are genius.	
	3) There is a student who likes Discrete Mathematics but not Probability and Statistics.	
	4) Each integer is either even or odd.	
	(c) Determine the validity of the argument given:	<b>07</b>
	If I study, then I will not fail in Discrete Mathematics.	
	If I do not play cricket, then I will study.	
	But I failed in Discrete Mathematics.	
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	Therefore I must have played cricket.	
<b>OR</b>		
(c)	Find if the following is a tautology, contradiction or contingency.	<b>07</b>
	$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$	
<b>Q.3</b>	(a) Define: Bounded, Distributive and Complemented Lattices.	<b>03</b>
	(b) Find the transitive closure of	<b>04</b>
	$R = \{(1,2), (3,4), (4,5), (4,1), (1,1)\}$ . Where, $A = \{1,2,3,4,5\}$ .	
(c) Let $A$ be a set of factors of positive integer $m$ and relation is divisibility on $A$ .	<b>07</b>	
	For $m = 45$ , show that POSET $(A, \leq)$ is a Lattice.	
<b>OR</b>		
<b>Q.3</b>	(a) Draw the Hasse diagram of the set $\{1,3,9,18\}$ under partial order relation 'divides' and indicate those which are chains.	<b>03</b>
	(b) Let $X = \{1,2,3, \dots, 7\}$ and $R = \{(x,y): x - y \text{ is divisible by } 3\}$ . Show that $R$ is an equivalence relation. Draw the graph of $R$ .	<b>04</b>

(c) Solve the recurrence relation: 07

$$a_{n+2} - 5a_{n+1} + 6a_n = 2.$$

**Q.4** (a) Define group with example. Give an example of a non-abelian group. 03

(b) Let  $H = \{[0], [3]\}$  in  $Z_6$  under addition. Find left and right cosets in  $\langle Z_6, +_6 \rangle$ . 04

(c) Prove that  $G = \{1,2,3,4,5,6\}$  is a finite abelian group of order 6 with respect to multiplication modulo 7. 07

**OR**

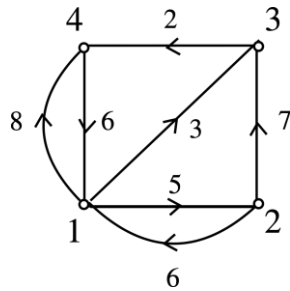
**Q.4** (a) Define subgroup and group Homomorphism. 03

(b) Is addition a binary operation on  $\{-1,0,1\}$ ? Justify. 04

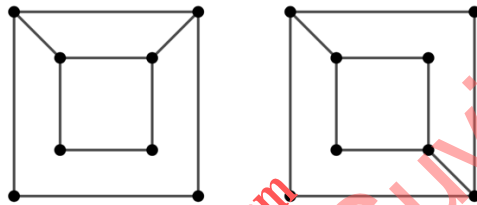
(c) Explain Cosets and Lagrange's theorem. 07

**Q.5** (a) How many nodes are necessary to construct a graph with exactly 8 edges in which each node is of degree 2. 03

(b) Find the shortest path between each pair of vertices for a simple digraph using Warshall's Algorithm. 04



(c) Define Isomorphic Graphs. Verify the following graphs are Isomorphic or not (Justify). 07



**OR**

**Q.5** (a) Define Cyclic graph, Null graph and Strongly connected graph. 03

(b) Draw a graph which is regular but not bipartite. 04

(c) For the following set of weights construct an optimal binary prefix code. For each weight in the set, give corresponding code word. 07  
5, 7, 8, 15, 35, 40

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